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The mathematical theory of finite and infinite dimensional topological spaces and its relevance to quantum gravity

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ABSTRACT

The present work gives first a review of the mathematical theory of finite and infinite dimensional topological spaces. Subsequently we connect the discussion with E-infinity theory and the theory of partially ordered sets. Finally, we contemplate the relevance of abstract results for quantum gravity.

1. Introduction

As noted by various authors, the mathematical theory of dimensions is still regarded by many as something isoteric with little if any relevance to physics [1–8]. Even those working in high energy physics with many extra dimensions do not seem in general to be fully aware of the power of the theories of Menger and Urysohn and its connection to the dimensional theory of Hausdorff [1–4].

In the present work we give first a short review of the results of the theory of finite and infinite dimensional topological spaces [1,2]. Subsequently we address various aspects of this theory in connection with E-infinity theory and the theory of partially ordered sets [8,9]. Finally, we look into the applications of these results in high energy physics and particularly quantum gravity [10–26].

2. Menger's present to Einstein – coordinate less and pointless geometry

Only a few weeks after Albert Einstein's death in 1955 the book edited by P.A. Schilpp "Albert Einstein als philosoph und Naturforscher" appeared [27]. This book contained a remarkable paper as a birthday present by the great Austrian-American topologist Karl Menger to Einstein [28]. The German title of the paper was "Die Relativitäts theorie und die Geometrie" which ponders the relations between theory of relativity and geometry. From all that we know, Einstein did not like the article that much. In fact the book was edited by Prof. R. Sexl and reprinted in 1983 without including this paper [27,28]. Before speculating about the reason for excluding this paper or why Einstein was not made about it, let us review the main message of this paper first.

Menger starts his paper with a historical review of the development of geometry beginning with antiquity and Euclid. He explained the importance of the coordinate system in Descartes work for the maritime transportation of the 17th century. The application of the calculus of differentiation and integration to this coordinate's geometry is then considered in the light of the work of Euler, Monge and Gauss. In the 19th century, one encounters the non-Euclidean geometry of Bolya

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leading to (4) $(\bar{\alpha}_\circ) \cong 548$ isometries. In addition it is shown in E-infinity that (5) $(\bar{\alpha}_\circ) \cong 685$ corresponds to Dim $E_{12} = 685$ as well as the sum of the dimensions of all the 17 two and three Stein spaces [10,19].

6. At the Planck length we do not have fractals. We have the multiple connectivity of Wheeler's spacetime foam, but this is not a fractal. At the Planck energy scale we have a deterministic non-fractal quasi classical universe as argued by El Naschie as well as the proponents of the Planck Aether hypothesis [10,37].
7. The work of Dowker, Sorkin, Henson, Ambjorn, Loll and Bombelli on posets should be seen in the light of the work of Hemion and El Naschie and the other way around [8,30,32].
8. The concepts foliation, fuzzy sets and fractals are exchangeable as far as the practical implications of quantum gravity are concerned. It seems, however, that E-infinity has inbuilt numerical machinery which leads to exact quantitave results without a computer. There is no doubt that this is a natural consequence of using the golden mean number system [10,19].
9. The sprinkling technique used by the proponents of causal set theory which is based on a random spacetime points selection using a Poisson distribution is just another variant of the gamma discrete distribution used by El Naschie in his E-infinity theory [10,19,33].
10. We could easily derive $D_T = 3 + 1 = 4$ and $D_H = 4 + \phi^3$ from a Hilbert cube in fractal spacetime. Let us nest a 4D cube inside a 4D cube and so on indefinitely. That way we find, using continued fraction, that $D_H = 4 + (\bar{4}) = 4 + \phi^3$. Next we move from fractal-Cantorian space with $d_c^{(0)} = (0.5) + 0.118033898$ to $\langle d_c^{(0)} \rangle = 0.5$. This way we find $\sim \langle n \rangle = (1 + 0.5)/(1 - 0.5) = 3$. Setting in the bijection formula [10] one finds $d^{(3)} = [1/(0.5)]^{3-1} = (2)^2 = 4$. Thus $D_T = 3 + 1 = 4$.
11. Although strictly speaking the Hausdorff dimension is neither a pure topological concept nor a pure topological invariant, it is deeply related to the inductive theory of Menger and Urysohn. El Naschie's extension of the Menger–Urysohn theory by proving that the dimension of the totally empty set is $-\infty$ is the deep connection.

In conclusion we express our belief that mathematical beauty may turn out to be far more important than Dirac ever suspected. Fractal spacetime for instance has vindicated Weyl's beautiful original gauging ideas, which are used in E-infinity theory. Similarly, the equation for the unification inverse coupling is of such elegance and simplicity that confinement becomes a trivial consequence [25]. This all could not have been possible without a serious study of the mathematical theory of dimensions.

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